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The enhancement of stimulated emission near a photonic band edge

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Abstract

We have derived an analytical expression for light amplification for stimulated emission in arbitrary two-dimensional photonic crystals (PHCs) using the response formula of the PHCs and the Bloch equations. This shows that the gain is proportional to the population inversion, and is inversely proportional to the group velocity and the relaxation coefficient of the phase, which indicates that the gain can be induced by population inversion and enhanced dramatically by small group velocity or anomalous group velocity.

1. Introduction

It has been demonstrated that the small group velocities of the eigenmode would bring about an enhancement of the effective gain or low-threshold lasing [1–4]. Konôpka [5] analysed stimulated emission (STE) from the atom when its particular transition is tuned near the band gap edge in a fully quantized field way. He described a model in which the initial multimode field is a superposition of single photon states with the spectrum of energy localizing near the band edge, and he assumed that the atom was at a fixed position. These assumptions have some limitations. His intention was to demonstrate the influence of the initial field on the dynamics of the atom–field system. He judged the STE near the band edge by whether the atom predominantly emits into the initially occupied modes, which needs a very complicated numerical calculation. The lasing threshold in photonic crystal (PHC) was also discussed by carrying out a multiscale analysis of the appropriate Maxwell–Bloch equations [6], in which the authors focused on band-edge lasers in one-dimensional (1D) and two-dimensional (2D) PHC by using the Maxwell–Bloch equation and numerical calculation. We study the optical gain and the lasing threshold by the formula for optical responses of the PHC and the Bloch equations. We derive that the gain is proportional to the population inversion, and is inversely proportional to the group velocity and the relaxation coefficient. This shows that there is an enhancement to STE due to the small group velocity. The physical meaning of the result obtained is explained.

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2. Analysis for light amplification

A periodic dielectric medium doped with two-level atoms is studied here. We consider that the light propagates in a 1D PHC or a 2D PHC. For intense optical pulses containing many photons a semi-classical treatment of the radiation field is adequate. We assume that the polarized direction is parallel to the periodic plane. The Maxwell–Bloch equations are used to describe the interacted-atom–field system. The propagation of electromagnetic radiation from an impurity atom embedded in a PHC can be described by the following equations:

$$\begin{aligned} &-\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}+\zeta\right)Q(\mathbf{r},t)=\frac{1}{c^2\varepsilon_0\sqrt{\varepsilon(\mathbf{r})}}\frac{\partial^2}{\partial t^2}P(\mathbf{r},t),\\ &Q(\mathbf{r},t)=\sqrt{\varepsilon(\mathbf{r})}E(\mathbf{r},t),\\ &\zeta\, Q(\mathbf{r},t)=\frac{1}{\sqrt{\varepsilon(\mathbf{r})}}\nabla\times\left\{\nabla\times\frac{1}{\sqrt{\varepsilon(\mathbf{r})}}Q(\mathbf{r},t)\right\}, \end{aligned} \tag{1}$$

where $P(\mathbf{r},t)$ is the polarization field of the impurity atoms induced by the external field, and $\varepsilon(\mathbf{r})$ is the dielectric constant of the material, which is a periodic function of the position $\mathbf{r}$. According to the result Sakoda obtained [7], the induced electric field can be expressed as

$$E(\mathbf{r},t)+\frac{P(\mathbf{r},t)}{\varepsilon_0\varepsilon(\mathbf{r})}=\frac{1}{\varepsilon_0 V}\sum_{\mathbf{k}'n'}E_{\mathbf{k}'n'}(\mathbf{r})\int_V \mathrm{d}\mathbf{r}\int_{-\infty}^{t}\mathrm{d}t'\,E^*_{\mathbf{k}'n'}(\mathbf{r})P(\mathbf{r},t')\omega_{\mathbf{k}'n'}\sin\omega_{\mathbf{k}'n'}(t-t'). \tag{2}$$

This is just the optical response formula of the PHC. Here $E_{\mathbf{k}n}(\mathbf{r})$ is an eigenmode propagating in the crystal, and V is the volume of the PHC.

On the other hand, the dynamics of the atoms in a PHC can be described by Bloch equations:

$$\frac{\mathrm{d}P(\mathbf{r},t)}{\mathrm{d}t}=(-\mathrm{i}\varpi-\gamma_P)P(\mathbf{r},t)+\frac{1}{\mathrm{i}\hbar}(E(\mathbf{r},t)d^2)D(\mathbf{r},t), \tag{3}$$

$$\frac{\mathrm{d}D(\mathbf{r},t)}{\mathrm{d}t}=-\gamma_D(D(\mathbf{r},t)-D_0(\mathbf{r},t))-\frac{4}{\mathrm{i}\hbar}E(\mathbf{r},t)P^*(\mathbf{r},t), \tag{4}$$

$$D(\mathbf{r},t)=\rho(\mathbf{r})D(t), \tag{5}$$

where $D(t)$ represents the population inversion, and is only a function of time t. $D(\mathbf{r},t)$ represents the population inversion at position $\mathbf{r}$, $D_0(\mathbf{r},t)$ is the steady-state equilibrium inversion at position $\mathbf{r}$, $\rho(\mathbf{r})$ characterizes the atom distribution within the PHC, i.e., the atoms' number density, d is the dipole matrix element of the atomic transition, ϖ is the atomic resonance frequency, and γ_D, γ_P is the relaxation coefficient of the population and that of the phase, respectively.

Letting $E=\tilde{E}(\mathbf{r},t)\mathrm{e}^{-\mathrm{i}\omega t}$, $P=\tilde{P}(\mathbf{r},t)\mathrm{e}^{-\mathrm{i}\omega t}$, and substituting them into equations (3) and (4), we can obtain

$$\frac{\mathrm{d}\tilde{P}(\mathbf{r},t)}{\mathrm{d}t}=(-\mathrm{i}(\varpi-\omega)-\gamma_P)\tilde{P}(\mathbf{r},t)+\frac{1}{\mathrm{i}\hbar}(\tilde{E}(\mathbf{r},t)d^2)D(\mathbf{r},t), \tag{6}$$

$$\frac{\mathrm{d}D(\mathbf{r},t)}{\mathrm{d}t}=-\gamma_D(D(\mathbf{r},t)-D_0(\mathbf{r},t))-\frac{4}{\mathrm{i}\hbar}\tilde{E}(\mathbf{r},t)\tilde{P}^*(\mathbf{r},t), \tag{7}$$

where $\tilde{P}(\mathbf{r},t)$ is the slow-variation amplitude of the polarization, $\tilde{E}(\mathbf{r},t)$ is the slow-variation amplitude of the electronic field, and ω is the frequency of the electric field. In the following, the wave signal is omitted.

Assuming $\gamma_P \gg \gamma_D$, the variation of the polarization can follow the variation of the electric field in the period of $1/\gamma_P$, so we can let $\mathrm{d}\tilde{P}(\mathbf{r},t)/\mathrm{d}t=0$ in equation (6), and then we

can obtain

$$P(\mathbf{r},t)=\frac{d^2}{(\omega-\varpi+\mathrm{i}\gamma_P)\hbar}E_{kn}(\mathbf{r},t)\rho(\mathbf{r})D(t). \tag{8}$$

When we consider the case that the eigenfrequency is resonant with the two-level atom, equation (8) can be expressed as

$$P(\mathbf{r},t)=\frac{d^2}{\mathrm{i}\gamma_P\hbar}\rho(\mathbf{r})D(t)E_{kn}(\mathbf{r})\exp\{(-\mathrm{i}\omega+\delta)t\}. \tag{9}$$

To assure adiabatic switching of the polarization, a positive infinitesimal δ is introduced, and γ_P is assumed to be a very small finite positive quantity. Substituting equation (9) into (2), the propagating part of the induced electric field is

$$E(\mathbf{r},t)=\frac{d^2D(t)}{2\varepsilon_0V}\frac{1}{\mathrm{i}\hbar\gamma_P}\sum_{\mathbf{k}'n'}\omega_{\mathbf{k}'n'}E_{\mathbf{k}'n'}(\mathbf{r})\int_V\mathrm{d}\mathbf{r}\,\rho(\mathbf{r})E^*_{\mathbf{k}n}(\mathbf{r})\int_{-\infty}^{t}\mathrm{d}t'\,E_{\mathbf{k}'n'}(\mathbf{r},t')$$
$$\times\left(\frac{1}{\omega_{\mathbf{k}'n'}+\omega+\mathrm{i}\delta}-\frac{1}{\omega-\omega_{\mathbf{k}'n'}+\mathrm{i}\delta}\right). \tag{10}$$

In the deduction, the structure of the PHC is assumed to be a cubic lattice, but the following result is applicable to arbitrary structure PHCs. Similarly to the analysis made by Sakoda [10], the total induced electric field of the impurity atoms can be approximated to

$$E(\mathbf{r},t)\approx E_{\mathbf{k}n}(\mathbf{r})\exp(\beta l)\exp(-\mathrm{i}\omega t), \tag{11}$$

where

$$\beta=\frac{f\omega_{kn}D(t)d^2}{2\varepsilon_0v_\mathrm{g}\hbar\gamma_P} \tag{12}$$

is the amplitude amplification factor of the induced electric field per unit length, l is the PHC length along the light propagating direction, and $f=\frac{1}{V_0}\int_{V_0}\mathrm{d}\mathbf{r}\,\rho(\mathbf{r})|\mathbf{E}_{\mathbf{k}n}(\mathbf{r})|^2$ is the effective number density of the impurity atoms.

3. Results and discussions

From equation (11), the gain of the STE intensity can be obtained:

$$G=2\beta=\frac{f\omega_{kn}D(t)d^2}{\varepsilon_0v_\mathrm{g}\hbar\gamma_P}. \tag{13}$$

On the other hand, we can deduce the population inversion $D(t)$ from equation (7). The steady-state process can determine many properties such as the laser threshold, the gain and the laser line width. When we consider the steady-state case, that is, $\mathrm{d}D(\mathbf{r},t)/\mathrm{d}t=0$, and $\mathrm{d}P(\mathbf{r},t)/\mathrm{d}t=0$, using equations (6), (7) and (5), we can get

$$D(\mathbf{r},t)=\frac{D_0(\mathbf{r},t)}{1+(|E(\mathbf{r},t)|^2/I_\mathrm{sat})}, \tag{14}$$

$$D(t)=\frac{D_0(t)}{1+(|E(\mathbf{r},t)|^2/I_\mathrm{sat})}, \tag{15}$$

where $I_\mathrm{sat}=\hbar^2\gamma_P\gamma_D/4d^2$ is the line-centre saturation field intensity (it represents the field intensity at which the nonlinear response of the atoms becomes important), and $D_0(t)$ is the steady-state equilibrium inversion. For a field intensity $I\gg I_\mathrm{sat}$, the atomic response is saturated. In a PHC, I_sat, which may be modified by the local density of state [6], is different from its value in free space.

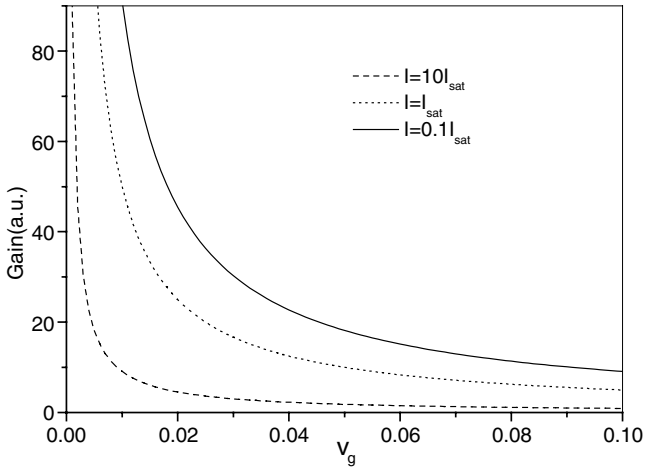

Figure 1. Normalized gain as a function of the group velocity v_g when the eigenmode intensity $I = |E(r, t)|^2 = 0.1I_{sat}$ (solid curve), $I = I_{sat}$ (dotted curve) and $I = 10I_{sat}$ (dashed curve). v_g is measured in units of $\alpha = \frac{f\omega_{kn}d^2}{\varepsilon_0\hbar\gamma_P}\frac{D_0(t)}{1+(|E(\mathbf{r},t)|^2/I_{sat})}$.

Substituting equation (15) into (13), we thus obtain

$$G = \frac{f\omega_{kn}d^2}{\varepsilon_0\hbar\gamma_P v_g}\frac{D_0(t)}{1+(|E(\mathbf{r},t)|^2/I_{sat})}. \tag{16}$$

The gain is proportional to the steady-state equilibrium inversion, but it is inversely proportional to the group velocity of the eigenmode and the relaxation coefficient of the phase. Given the group velocity, the steady-state equilibrium inversion and the relaxation coefficient, the optical gain of STE in an arbitrary PHC can be calculated by equation (16). This indicates that the gain in a PHC could be induced by the external field (reflected by population inversion), and then can be enhanced by the small group velocity in the PHC. Near the band gap edge of a PHC with infinite size, the group velocity approaches zero [8], and then the gain will become infinite, which is described in figure 1. Furthermore, some scholars have demonstrated that the group velocity anomaly can occur on the pass photonic band [4], which will cause a fairly large enhancement of STE. Dowling *et al* [8] showed that the enhancement comes from the long interaction time between the radiation field and the matter system due to the small group velocity. Near the band gap edge or at the pass band where the group velocity is anomalistic, the small group velocity is the dominant contribution to the gain, no matter how small the population inversion $D_0(t)$ is. In particular, the gain can even occur when there is no population inversion [9]. On the other hand, if the group velocity is not small, such as at some pass band, but the population inversion is large enough, the gain is mainly induced by the population inversion. The above results are different from that in the uniform material where the optical gain will only be induced by population inversion in the general case. Moreover, in uniform material, when the electric field intensity $|E(\mathbf{r}, t)|^2 > I_{sat}$, the optical gain begins to saturate. But in a PHC, even when $|E(\mathbf{r}, t)|^2 \gg I_{sat}$, the gain may still be enhanced largely due to the small group velocity. It can be found from figure 1 that, for a constant group velocity, the optical gain begins to saturate under the condition that $|E(\mathbf{r}, t)|^2 > I_{sat}$; therefore, the gain still increases with decreasing group velocity.

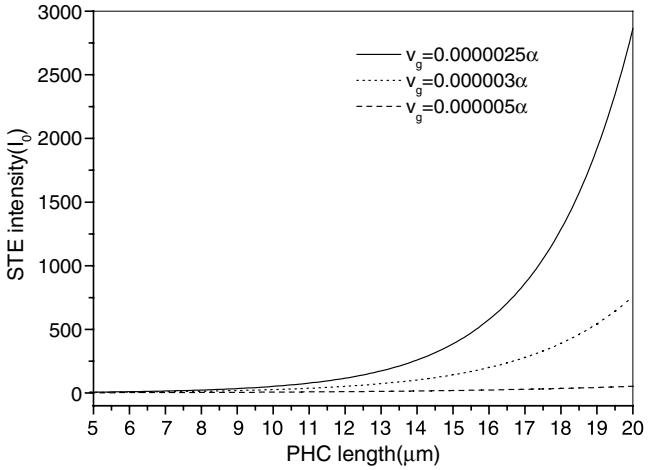

Figure 2. STE intensity as a function of the PHC length with group velocity $v_g = 2.5 \times 10^{-6}$ (solid curve), $v_g = 3.0 \times 10^{-6}$ (dotted curve) and $v_g = 5.0 \times 10^{-6}$ (dashed curve). v_g is measured in units of $\alpha = \frac{f\omega_{kn}d^2}{\varepsilon_0\hbar\gamma_P}\frac{D_0(t)}{1+(|E(\mathbf{r},t)|^2/I_{sat})}$.

From equations (11) and (15), the optical intensity of STE can be obtained:

$$I = I_0 \exp\left(\frac{f\omega_{kn}d^2}{\varepsilon_0\hbar\gamma_P v_g}\frac{D_0(t)}{1+(|E(\mathbf{r},t)|^2/I_{sat})}l\right), \tag{17}$$

where I_0 represents the initial field intensity. When the population inversion and the group velocity of the PHC are appropriate values, the STE will increase exponentially with increasing PHC length (see figure 2). The STE intensity is extremely sensitive to v_g, $D_0(t)$ and l. For example, assuming $\alpha = \frac{f\omega_{kn}d^2}{\varepsilon_0\hbar\gamma_P}\frac{D_0(t)}{1+(|E(\mathbf{r},t)|^2/I_{sat})}$, with $l = 50\ \mu$m, $v_g = 5.0 \times 10^{-6}\alpha$, $I = 53.5I_0$ and for $v_g = 2.5 \times 10^{-6}\alpha$, $I = 2864I_0$. This phenomenon also demonstrates that the STE can be drastically enhanced when the group velocity is very small. The STE intensity can be evaluated practically by using equation (17), because the other parameters in equation (17), such as I_0, $D_0(t)$, γ_P, I_{sat}, f, and d etc, can be determined by the experimental conditions. This equation will benefit the design of a photonic band gap edge laser.

As an example, we proceed to the quantitative evaluation of the enhancement of STE for a 2D PHC with a square lattice. Figure 3(a) shows the transmittance as a function of the frequency for an incident wave with S polarization propagated perpendicular to the 2D PHC with dielectric constant of 11.4. Figure 3(b) shows the transmittance for S polarization calculated for the same crystal as figure 3(a) with a dielectric constant of $11.4 - 0.02$i. The crystal consists of 153 (9×17) dielectric rods arranged as a square lattice with lattice constant 250 nm and rod radius 74 nm. The negative imaginary part of the dielectric constant represents the inverted population of the impurity atoms [11], corresponding to $D_0(t)$ in equation (17). Note that the transmittance can be greater than unity because of the STE that takes place in the crystals. Here, the strongest peak shows a large enhancement of the STE at 765 THz, which exactly coincides with the third upper band-edge frequency (see figure 3(a)) where $v_g = 0$. This phenomenon simply demonstrates the result of equation (17). The enhancement factor, which was estimated as the ratio of the transmittance when there are impurity atoms (shown in figure 3(a)) to that when there are no impurity atoms (shown in figure 3(b)) in the PHCs, is as large as 141. Because the calculated PHC is far from the infinite crystal, it is quite reasonable

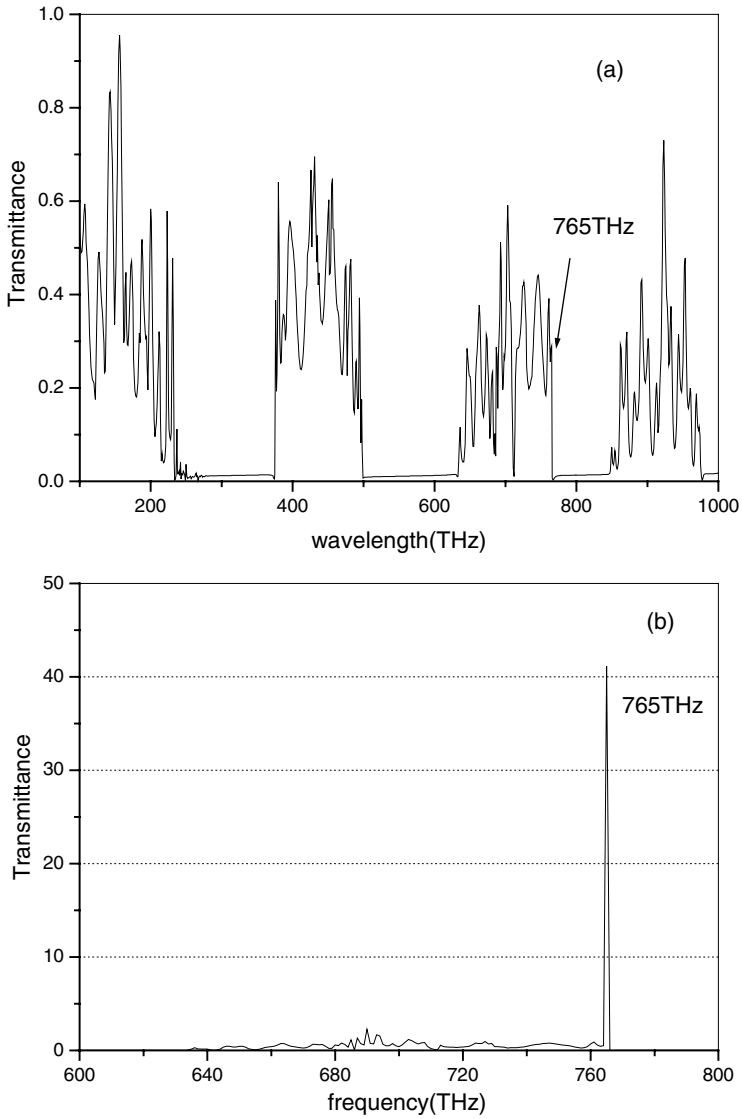

Figure 3. (a) The transmittance for S polarization as a function of the frequency calculated for a square lattice crystal with 153 dielectric rods with dielectric constant of 11.4; (b) the transmittance for S polarization as a function of the frequency calculated for the same crystal as (a) with dielectric constant of $11.4 - 0.02$i. The negative imaginary part of the dielectric constant represents the population inversion of the impurity atoms. Note that the transmittance can be greater than unity because of the STE.

that the amplification factor is not infinite due to the fact that $v_g = 0$ at the band edges may be difficult to attain with a finite size crystal.

4. Conclusion

In summary, we derive a practical formula for the light amplification in arbitrary structure 2D PHCs by means of the Maxwell–Bloch equation. The optical gain in the PHCs is proportional

to the population inversion and is inversely proportional to the group velocity. This shows that the gain can be induced by population inversion and be enhanced by the small group velocity. The difference between the STE in the PHCs and that in uniform materials is analysed.

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